Code No: 111AL **R13**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year Examinations, January/February - 2024

MATHEMATICAL METHODS

(Common to EEE, ECE, CSE, IT)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Write the normal equations to fit the parabola $y = a + bx + cx^2$ using least squares method. [2]

b) Determine $(2\Delta+3)(E+2)(3x^2+2)$ by taking $h=1$. [3]

c) Explain Trapezoidal to find the approximate value of $\int_a^b f(x)dx$. [2]

d) Derive the recurrence relation to evaluate $\sqrt{N}$ using Newton Raphson's method. [3]

e) Is $f(x) = \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x$ periodic? If so, find its period. [2]

f) Obtain the Fourier sine integral of $f(x) = e^{-kx}$, $k > 0$ & $x \geq 0$. [3]

g) Using the method of separation of variables, find solution of $x^2\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. [2]

h) Form a partial differential equation by eliminating arbitrary constants a, b from $2z = \sqrt{(x+a)} + \sqrt{(y-a)} + b$. [3]

i) For what value(s) of *p*, the vector filed $\bar{f} = (x+3y)\bar{i} + (y-2z)\bar{j} + (x+pz)\bar{k}$ is solenoid vector? [2]

j) Compute the work done in moving a particle in the force field $\bar{F} = 3x^2\bar{i} + \bar{j} + z\bar{k}$ along the straight line from $(0,0,0)$ to $(2,1,3)$. [3]

PART – B

(50 Marks)

2.a) Prove or disprove: $\mu^2 = 1 + \frac{\delta^2}{4}$.

b) Use Lagrange's interpolation formula to fit a polynomial to the following data.

x	-1	0	2	3
y	-8	3	1	2

Hence find $y(-2)$, $y(1)$ and $y(4)$. [4+6]

OR

3.a) Explain the method of least squares.

b) Determine the constants a and b by the method of least squares such that $y = a e^{bx}$.

[3+7]

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

4.a) Compute a positive real root of the equation $x \log_{10} x = 1.2$ correct to three decimal places using Bisection method.

b) Using the Gauss Seidel iterative method, solve:
$5x - y = 9;\ x - 5y + z = -4;\ y - 5z = 6$. [5+5]

OR

5. Use a) Taylor series method b) Runge-Kutta method of 4th order to solve $10\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1$ for the interval $0 < x \le 0.2$ with $h = 0.1$. [5+5]

6. Find the Fourier series of periodic function f(x) of period 2, where $f(x) = \begin{cases} -1 & if\ \ -1 < x < 0 \\ 2x & if\ \ \ 0 < x < 1 \end{cases}$. Hence, prove that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{4}$. [10]

OR

7.a) State the Fourier integral theorem.

b) Represent $f(x)$ as an exponential Fourier transform when $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$.

Show that the result can be written as $f(x) = \frac{1}{\pi}\int_0^\infty \frac{\cos \lambda x - \cos \lambda(x - \pi)}{1 - \lambda^2} d\lambda$. [2+8]

8. Solve the following differential equations

a) $(z^2 - 2yz - y^2)p + (xy + xz)q = xy - xz$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

b) $(p^2 + q^2)y = qz$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. [4+6]

OR

9. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\sin(\pi x/l)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$ [10]

10.a) Calculate the scalar potential ϕ such that $\bar{f} = \nabla\phi$ if the vector field $\bar{f} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is an irrotational vector.

b) Evaluate the directional derivative of $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). [5+5]

OR

11. Verify the divergence theorem for $\bar{f} = (x^2 - yz)\,\bar{i} + (y^2 - zx)\,\bar{j} + (z^2 - xy)\,\bar{k}$ taken over the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. [10]

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